

ALGEBRAIC TOPOLOGY I WS23/24, HOMEWORK SHEET 8

DEADLINE: FRIDAY, DECEMBER 8TH

Problem 1. Show the following for every $n \geq 0$: If there is a weak homotopy-equivalence

$$\Sigma^n \mathbb{R}P^\infty \simeq A \vee B,$$

then A or B is weakly contractible.

(If you are interested, you can also contemplate the following question for various values of k and n : Is $\Sigma^n \mathbb{R}P^k$ weakly equivalent to a wedge sum of two spaces? But don't expect to obtain a full answer!)

Problem 2. Reprove the Freudenthal suspension theorem stated below by induction on n , making use of transgressions in Serre spectral sequences. (Hint: Rephrase the problem in terms of connectivity of the map $X \rightarrow \Omega \Sigma X$ and apply the Whitehead/Hurewicz theorem).

Theorem(Freudenthal). Suppose that X is an $(n - 1)$ -connected space for some $n \geq 2$. Then the suspension homomorphism $\Sigma_* : \pi_k(X, *) \rightarrow \pi_{k+1}(\Sigma X, *)$ is an isomorphism if $k < 2n - 1$ and an epimorphism if $k = 2n - 1$.